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A new possible explanation for why a neutron decays

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Abstract

If the neutron decay time is a "constant," we propose a new approximate equation and then attempt to interpret it under this assumption.

Keywords: Neutron decay, Neutron mass, Proton mass, Interaction

1 Assuming that the neutron decay time is a 'constant'

Assuming neutron decay time: $(\tau) = (2\pi)^3 \frac{(2)(a_0)(R_\infty)(\Delta(m_m))}{(h)} = (2\pi)\frac{[\alpha_0](\Delta(m_m))}{(\hbar)}$.

1.1 A possible calculation method for neutron decay time

In order to generate this decay time formula for the modified Schr ö dinger equation:$(\tau) = (2\pi)^3 \frac{(2)(a_0)(R_\infty)(\Delta(m_m))}{(h)} = (2\pi)\frac{[\alpha_0](\Delta(m_m))}{(\hbar)}$. Starting from the Schr ö dinger equation itself, we can introduce an "effective Schr ö dinger equation":$i(\hbar)\frac{\partial}{\partial t}\psi = \left[-\frac{(\hbar)^2}{2(m_e)}\nabla^2 + V - i\frac{(\hbar)}{2(\tau)}\right]\psi$. Among them, Vcan be arbitrary, but we can also derive the Schr ö dinger equation that effectively includes the complex potential from the Lagrangian. Through an "equivalence principle", we introduce the dissipation function into the variation to obtain the equation of motion, and then map it to the Schr ö dinger equation under a slowly varying approximation.

We can also consider the specific forms of "no reflection potential" or "complex potential", for example, taking $V = 0$, the equation becomes $i(\hbar)\partial_t\psi = -\frac{(\hbar)^2}{2(m_e)}\nabla^2\psi - i\frac{(\hbar)}{2(\tau)}\psi$. The solution of this equation is

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$\psi \sim e^{-iEt/(\hbar)}e^{-t/(2(\tau))}$, normalized to volume V, the probability decay decay time is (τ). Substituting (τ), the imaginary part is $-\frac{i}{2}(\hbar)\frac{(\hbar)}{(2\pi)[\alpha_0](\Delta(m_m))}$.

Sort these out haphazardly, and the final order is to introduce a imaginary part of the complex potential into the Schr ö dinger equation, whose magnitude is derived from the (τ) . The specific method is as follows.

1. Primitive (non dissipative) Schr ö dinger equation: $i(\hbar)\frac{\partial\psi}{\partial t} = \left[-\frac{(\hbar)^2}{2(m_e)}\nabla^2 + V(\mathbf{r})\right]\psi$.

2. Modify to complex potential Schr ö dinger equation, introduce a constant complex potential: $V_{\text{eff}}(\mathbf{r}) = V(\mathbf{r}) - i\frac{(\hbar)}{2(\tau)}$, Then the equation becomes: $i(\hbar)\frac{\partial\psi}{\partial t} = \left[-\frac{(\hbar)^2}{2(m_e)}\nabla^2 + V(\mathbf{r}) - i\frac{(\hbar)}{2(\tau)}\right]\psi$.

3. Substitute into the target decay time formula, substitute $(\tau) = (2\pi)\frac{[\alpha_0](\Delta(m_m))}{(\hbar)}$ into the imaginary part: $\frac{(\hbar)}{2(\tau)} = \frac{(\hbar)}{2}\cdot\frac{(\hbar)}{(2\pi)[\alpha_0](\Delta(m_m))}$, therefore, the complex potential can be explicitly written as:$V_{\text{eff}}(\mathbf{r}) = V(\mathbf{r}) - i\,\frac{(\hbar)^2}{(4\pi)[\alpha_0](\Delta(m_m))}$, the Schr ö dinger equation becomes: $i(\hbar)\frac{\partial\psi}{\partial t} = \left[-\frac{(\hbar)^2}{2(m_e)}\nabla^2 + V(\mathbf{r}) - i\,\frac{(\hbar)^2}{(4\pi)[\alpha_0](\Delta(m_m))}\right]\psi$.

4. Prove consistent decay time, if $V(\mathbf{r})$ is independent of time, then the solution of the equation has the form: $\psi(\mathbf{r},t) = u(\mathbf{r})\,e^{-iEt/(\hbar)}\,e^{-t/(2(\tau))}$, among them, $u(\mathbf{r})$ satisfies the steady-state equation:$\left[-\frac{(\hbar)^2}{2(m_e)}\nabla^2 + V(\mathbf{r})\right]u(\mathbf{r}) = E\,u(\mathbf{r})$, therefore, the probability density varies over time as follows:$|\psi(\mathbf{r},t)|^2 = |u(\mathbf{r})|^2\,e^{-t/(\tau)}$, thus, the decay time is exactly: $(\tau) = (2\pi)\frac{[\alpha_0](\Delta(m_m))}{(\hbar)}$.

2 How is the calculation method for neutron decay time obtained

Since there is no problem with following this deduction logic, how did the result of neutron decay time come about? Of course, this is also a matter of speculation before the result. That is to say, I believe that protons or proton masses are stable structures, and neutrons or neutron masses are perfect structures. So, is their difference or quotient also fixed? After my wild imagination, I discovered, or rather, I thought that the mass difference between neutrons and protons $(\Delta(m_m))$ and neutron decay time (τ) could be represented by the numerical product of physical constant symbols.

That is,$(m_n) - (m_p) = (\Delta(m_m)) \approx \frac{(m_e)}{(4\pi)} \times \frac{1}{(4\pi)(\varepsilon_0)(c)}$, $2.5(m_e) \approx 2.3(m_e)$.

That is, $(\tau) \approx (\frac{1}{(4\pi)(\varepsilon_0)(c)})^2$, $880 \approx 900$.

Then we crudely discovered that without considering the dimension represented by the symbol of the

physical constant, and assuming that the dimension and numerical value represented by the symbol of the physical constant are viewed separately, there are several equations for neutron decay time that ignore approximate symbols when only looking at numerical values.

$$1.(\tau) = (4\pi)^2 \frac{(\Delta(m_m))^2}{(m_e)^2} = (2\pi)^2 \frac{(\Delta(m_m))^2(c)^4}{(m_n)}.$$

$$2.(\tau) = (2\pi)^3 \frac{(2)(a_0)(\Delta(m_m))}{(m_e)(G_N)} = (2\pi)^3 \frac{(2)(a_0)(R_\infty)(\Delta(m_m))}{(h)}.$$

$$3.(\tau) = (2\pi) \frac{(\Delta(m_m))}{(m_e)(a_0)(c)} = (2\pi)^2 \frac{(\Delta(m_m))}{(m_e)(2\pi)(a_0)(c)}.$$

$$4.(\tau) = (2\pi) \frac{(\Delta(m_m))}{(m_e)(R_\infty)\frac{(e_0)}{(4\pi)(\varepsilon_0)}} = (2\pi) \frac{(G_N)(\Delta(m_m))}{(h)\frac{(e_0)}{(4\pi)(\varepsilon_0)}}.$$

$$5.\frac{(2\pi)}{(c)(\tau)}(G_N) = \frac{(h)}{(\Delta(m_m))(c)} \frac{(e_0)}{(4\pi)(\varepsilon_0)}.$$

$$6.\frac{(2\pi)^2(r)^2}{(c)(\tau)} = \frac{(h)}{(\Delta(m_m))[\alpha_0](c)}.$$

A very interesting idea is that I had originally planned to write about six new possible methods for calculating neutron decay times, but halfway through, I discovered something that using the modified Schr ö dinger equation is very aesthetically pleasing, or rather, feels very natural, just like using the only square equation here to fit the Lagrange equation, it looks very natural.

So, is there any inherent connection here? And I did something very interesting, which is to provide this possible connection.

Of course, if there is a genius who has a sudden idea while calculating Compton scattering and analogizes the mass difference between neutrons and protons to a circle or the trajectory of a circle in the cycloid formula, would he think of anything special about the wavelength we calculate when the velocity is at a special value? If this genius realizes that there is something special about this wavelength and relates it to neutron decay or neutron decay time, then he may be able to obtain $\frac{(2\pi)^2(r)^2}{(c)(\tau)} = \frac{(h)}{(\Delta(m_m))[\alpha_0](c)}$. But there are too many confusing terms in the real world, and ultimately no one can think of this thing.

Given that the modified Schr ö dinger equation is non unitary, if it holds true, what should be the unitary evolution equation of its closed system? If it is simpler, what should be the unitary evolution equation of its simplest closed system? Interestingly, if it involves complex numbers, what should be the unitary evolution equation of its simplest complex space closed system? If you have a certain mathematical and physical foundation, the simplest way is to ask artificial intelligence.

It is interesting that if we use the approximate solution obtained from the unitary evolution equation

of a closed system, we will obtain the decay rate $(\Gamma) = \frac{1}{(\tau)} = (2\pi)|\Omega|^2\rho(E_0)$. If we do not find anything special, we may say that this is another ordinary assumption.

However, in order to ultimately obtain the solution to our modified Schr ö dinger equation, the condition must be $(\tau) = (2\pi)^3\frac{(2)(a_0)(R_\infty)(\Delta(m_m))}{(h)} = (2\pi)^2\frac{[\alpha_0](\Delta(m_m))}{(h)} = (2\pi)\frac{[\alpha_0](\Delta(m_m))}{(\hbar)}$.

That is to say $|\Omega|^2\rho(E_0) = \frac{(\hbar)}{(2\pi)^2[\alpha_0](\Delta(m_m))} = \frac{h}{(2\pi)^3[\alpha_0](\Delta(m_m))}$。

That is,， $(2\pi)^3|\Omega|^2\rho(E_0) = \frac{h}{[\alpha_0](\Delta(m_m))}$.

That is,$(2\pi)^2(\Delta(m_m))|\Omega|^2\rho(E_0) = (m_e)(c)(a_0)$.

Then we reduce the dimensionality to ， $(2\pi)^2(r)^2(\Delta(m_m))|\Omega|^2\rho(E_0) = (m_e)(c)(a_0)$.

Then, ignoring the dimensionality, consider another interesting aspect, which is that $(2\pi)(\Gamma)(G_N) = \frac{(h)}{((\Delta(m_m))}\frac{(e_0)}{(4\pi)(\varepsilon_0)}$.

When I wrote out this equation and included it in the inference process of neutron decay, I felt that it might be correct because the equation at least told me that it provided a decay environment.

So, how did the calculation method for neutron decay time come about? The answer is guessed. So, is it reasonable? The answer is that, at least for now, the process I provided is not reasonable. However, when it deals with interactions, you haven't been touched at all. I think when the real answer is presented to you, but conflicts with what you already know and consider to be partially correct, I don't think you can distinguish whether this real answer is right or not.

2.1 A calculation equation that conforms to neutron decay time

So, now back to the starting point of our story,$(m_n) - (m_p) = (\Delta(m_m))$, if we consider neutrons to be a perfect structure and protons to be a stable structure, then the difference between them can be calculated, and mathematically, "closed systems, unitary evolution" is also something that can provide a derivation process. That is to say, considering neutron decay as the simplest two-level model, it can be first thought of as a very simplified "neutron state decay state" model.

Then, the fact is that true neutron decay is not a strictly two-level system, as the decaying protons, electrons, and antineutrinos have countless different combinations of momentum and energy. The two-level model can only be used to understand why the probability of neutron states decreases, and cannot

accurately calculate the true neutron decay time.

However, according to this understanding, neutron decay is a strictly two-level system, and this two-level model can be used to understand why the probability of neutron states decreases and accurately calculate the true neutron decay time. Of course, at this point, you need to understand what the real neutron decay time you are calculating is.

So, now let's reverse engineer. If we assume $(\tau) = (2\pi)\frac{[\alpha_0](\Delta(m_m))}{(\hbar)} = \frac{1}{(\Gamma)}$, then from the simplest perspective, the equation can be:$i(\hbar)\frac{\partial\psi}{\partial t} = \left[E_S - i\,\frac{(\hbar)(\Gamma)}{2}\right]\psi$, That is the equation we provided above, but now, this is a non unitary non closed system.

Then, if we reverse again, we hope to obtain a closed unitary evolution equation that, after eliminating the environment, can yield the original equation. And we know that the simplest strictly closed unitary model is the two-dimensional complex space model: $\mathbb{C}^2$; The simplest closed unitary model that can approximate irreversible exponential decay is "a discrete system state plus a continuous spectral environment": $\mathbb{C} \oplus L^2(\mathbb{R})$.

So, now, the simplest closed model may be $\mathcal{H}_{\text{tot}} = \mathbb{C} \oplus L^2(\mathbb{R})$, which is "a system state" plus "an environment with a continuous energy spectrum". Its total Hamiltonian can be written as:$H_{\text{tot}} = E_S|S\rangle\langle S| + \int dk\, E_k|k\rangle\langle k| + \int dk\, [g_k|S\rangle\langle k| + g_k^*|k\rangle\langle S|]$, The overall state can be written as: $|\Psi(t)\rangle = a(t)|S\rangle + \int dk\, b_k(t)|k\rangle$, and its unitary evolution equation is:$i(\hbar)\frac{d}{dt}|\Psi(t)\rangle = H_{\text{tot}}|\Psi(t)\rangle$.

Under continuous spectrum, weak coupling, and Markov approximation, the system amplitude satisfies: $i(\hbar)\dot{a}(t) = \left[E_S - i\frac{(\hbar)(\Gamma)}{2}\right]a(t)$.

That is, $a(t) = a(0)e^{-iE_S t/(\hbar)}e^{-(\Gamma)t/2}$, $|a(t)|^2 = |a(0)|^2 e^{-(\Gamma)t}$.

2.2 An effective equation that is completely closed and strictly unitary

Okay, it's mixed with a lot of imperceptible things. Now let's take a closer look at this thing.

The above paragraph is describing a very simple "quantum system+environment" model. The core meaning is that a quantum state can either stay in the system first or escape into the environment through interactions; There are many possible energy states in the environment.

If the environment has only one state, or a finite number of states, the amplitude will exchange back

and forth between the system and the environment, usually resulting in oscillation and reflux. However, when the environment has a continuous spectrum, the amplitude leaked out by the system will be dispersed into a large number of environmental states with different energies.

Each environmental state has a different phase: $e^{-iE_k(t-t')/(\hbar)}$, and over time, the phases corresponding to different energies shift and cancel each other out. The leaked amplitude is difficult to return neatly to the system state. Therefore, from a system perspective, probability is like a "one-way inflow" environment.

However, if the continuous environment absorbs the amplitude of the system, under weak coupling and memoryless approximation, this loss becomes a constant rate exponential decay, and the reduced dynamics of the complete Hermitian Hamiltonian manifest as a non Hermitian effective Hamiltonian with imaginary parts. And the imaginary part is not added out of thin air. It continuously transmits probability amplitudes from the system state to the continuous environmental state, so when we no longer inquire about the details of the environment, this outflow manifests as attenuation. Therefore, when we only look at the system part, we will obtain an effective equation with imaginary parts.

So, first we assume that the total state is: $|\Psi(t)\rangle = a(t)|S\rangle + \int dk\, b_k(t)|k\rangle$, where $a(t)$ is the probability amplitude of the system state $|S\rangle$; $b_k(t)$ is the probability amplitude of the k continuous state $|k\rangle$ in the environment; At the beginning, if the particles are completely in the system, then: $a(0) = 1$, $b_k(0) = 0$.

The complete Hamiltonian is:$H_{\text{tot}} = E_S|S\rangle\langle S| + \int dk\, E_k|k\rangle\langle k| + \int dk\, [g_k|S\rangle\langle k| + g_k^*|k\rangle\langle S|]$, Among them, E_S is the system state energy; E_K is the energy of the environmental state; g_k is the coupling strength between the system and the environmental state. The total evolution strictly follows: $i(\hbar)\frac{d}{dt}|\Psi(t)\rangle = H_{\text{tot}}|\Psi(t)\rangle$, and the conservation of total probability:$|a(t)|^2 + \int dk\, |b_k(t)|^2 = 1$.

Now substitute the total state into the Schr ö dinger equation and compare the coefficients before the system state and the environment state separately. The results are: $i(\hbar)\dot{a}(t) = E_S a(t) + \int dk\, g_k b_k(t)$ and $i(\hbar)\dot{b}_k(t) = E_k b_k(t) + g_k^* a(t)$. The first equation represents that the system amplitude not only evolves according to its own energy, but also receives environmental feedback. The second equation represents that every state in the environment is continuously 'injected' with amplitude by the system state.

That is to say, if the second equation can solve for the environmental amplitude, then first deal with the second equation: $i(\hbar)\dot{b}_k(t) = E_k b_k(t) + g_k^* a(t)$. Under the initial condition $b_k(0) = 0$, its solution is:

$b_k(t) = -\frac{i}{(\hbar)} g_k^* \int_0^t dt'\, e^{-iE_k(t-t')/(\hbar)} a(t')$. The meaning of this sentence is: at time t' , the system transitions to the environmental state $|k\rangle$ with a slight amplitude; After this amplitude, it evolves according to the energy E_k of the environment itself, and finally together forms the environmental amplitude at time t .

Then substitute the solution of the environmental amplitude back into the system equation, that is, substitute the $b_k(t)$ above into the first equation: $i(\hbar)\dot{a}(t) = E_S a(t) + \int dk\, g_k b_k(t)$, to obtain :$i(\hbar)\dot{a}(t) = E_S a(t) - \frac{i}{(\hbar)} \int_0^t dt' \int dk\, |g_k|^2 e^{-iE_k(t-t')/(\hbar)} a(t')$, It can also be written as:$\dot{a}(t) = -\frac{iE_S}{(\hbar)} a(t) - \int_0^t dt'\, K(t - t')a(t')$, where the memory kernel is:$K((\tau)) = \frac{1}{(\hbar)^2} \int dk\, |g_k|^2 e^{-iE_k(\tau)/(\hbar)}$, At this point, no approximation has been made and the complete system is still unitary, but you will see that the system now depends on $a(t')$ at all past moments. This is called the 'memory effect'.

Now let's assume, or rather, we have a Markov approximation, whose core assumption is that the correlation time of the environment is short and the system amplitude changes slowly. Therefore, within a short time interval where the memory kernel is significantly non-zero, it is possible to approximate $a(t') \approx a(t)$ and extend the original upper limit of integration from $t$to infinity: $\int_0^t dt' \longrightarrow \int_0^\infty dt'$. This way, the system no longer relies on detailed past history, but only on the current $a(t)$. This is the approximation of 'no memory'.

For easier visualization of the results, define the frequency as $\omega_S = \frac{E_S}{(\hbar)}$, $\omega_k = \frac{E_k}{(\hbar)}$, and write: $a(t) = c(t)e^{-i\omega_S t}$. Under weak coupling conditions, $c(t)$ changes slowly over time. After Markov approximation, we obtain: $\dot{c}(t) = -\left(\frac{(\Gamma)}{2} + i\Delta\omega\right) c(t)$, where $(\Gamma) = \frac{(2\pi)}{(\hbar)} \int dk\, |g_k|^2 \delta(E_S - E_k)$, and $\Delta\omega$ is the energy correction caused by the environment, also known as energy level shift or Lamb shift type correction.Among them, if the density of states of the environment is defined as $\rho(E) = \int dk\, \delta(E - E_k)$, and the coupling is approximated as a constant around E_S, then : $(\Gamma) = \frac{(2\pi)}{(\hbar)} |g(E_S)|^2 \rho(E_S)$, this is the Fermi Golden Rule.

Meanwhile, this is also the decay rate$(\Gamma) = \frac{1}{(\tau)} = (2\pi)|\Omega|^2 \rho(E_0)$ that we mentioned earlier. If the environmental channel has a continuous spectrum and ϕ is in an "empty" state at the initial time, then under the Wigner Weisskopf approximation, after tracing the ϕ degrees of freedom, ϕ satisfies:$i(\hbar)\frac{\partial \psi}{\partial t} = \left[E_S - i\,\frac{(\hbar)(\Gamma)}{2}\right]\psi$, The decay rate is: $(\Gamma) = (2\pi)|\Omega|^2 \rho(E_0)$. $|\Omega|^2$ describes the strength of the transition coupling between the system and the environment, that is, how strong the interaction is. $\rho(E_0)$ describes the density of states of the environment near the energy E_0, that is, how many channels there are for

particles to reach the final state of transition.

Now we multiply the phase factor back and get:$a(t) = a(0)\exp\left[-\frac{i}{(\hbar)}(E_S+\Delta E)t\right]\exp\left[-\frac{(\Gamma)t}{2}\right]$, Among them, $\Delta E = \hbar\Delta\omega$. If the energy correction ΔE is temporarily ignored, then $a(t) = a(0)e^{-iE_St/(\hbar)}e^{-(\Gamma)t/2}$. Therefore, $|a(t)|^2 = |a(0)|^2e^{-(\Gamma)t}$. So, the amplitude decays at a rate of $(\Gamma)/2$; Probability decays at a rate of (Γ) ; The probability decay time is: $(\tau) = \frac{1}{(\Gamma)}$.

Now this solution is equivalent to the effective equation: $i(\hbar)\dot{a}(t) = \left[E_S + \Delta E - i\frac{(\hbar)(\Gamma)}{2}\right]a(t)$, where the imaginary part is $-i\frac{(\hbar)(\Gamma)}{2}$, meaning that the probability of the system state flows towards the environment. It does not mean that the total probability disappears. The complete system still satisfies: $|a(t)|^2 + \int dk\,|b_k(t)|^2 = 1$. The non Hermitian effective equation simply states that when you only observe the system state and ignore the environmental state, the system part is no longer closed.

This is also what we mentioned above. When we no longer inquire about the details of the environment, this outflow manifests as attenuation. Therefore, when we only look at the system part, we will obtain an effective equation with imaginary parts.

And now we assume that the effective imaginary potential is $-i\frac{(\hbar)^2}{(4\pi)[\alpha_0](\Delta(m_m))}$, while the imaginary potential of the general effective equation is : $-i\frac{(\hbar)(\Gamma)}{2}$. After comparing the two, we find that $\frac{(\hbar)(\Gamma)}{2} = \frac{(\hbar)^2}{(4\pi)[\alpha_0](\Delta(m_m))}$, Therefore, $(\Gamma) = \frac{(\hbar)}{(2\pi)[\alpha_0](\Delta(m_m))}$.

And the system amplitude is $a(t) = a(0)e^{-iE_St/(\hbar)}\exp\left[-\frac{(\hbar)t}{(4\pi)[\alpha_0](\Delta(m_m))}\right]$, What is the probability that the system still exists $P_S(t) = |a(t)|^2 = |a(0)|^2\exp\left[-\frac{(\hbar)t}{(2\pi)[\alpha_0](\Delta(m_m))}\right]$, The corresponding decay time is $(\tau) = \frac{1}{(\Gamma)} = \frac{(2\pi)[\alpha_0](\Delta(m_m))}{(\hbar)}$.

So, this specific solution actually means that the system state has energy E_S and leaks to the continuous environmental channel at an attenuation rate (Γ); The system amplitude decays by $e^{-(\Gamma)t/2}$, while the probability in the system decays by $e^{-(\Gamma)t}$.

In a complete closed system: $|\Psi(t)\rangle = a(t)|S\rangle + \int dk\,b_k(t)|k\rangle$, total probability is always conserved: $|a(t)|^2 + \int dk\,|b_k(t)|^2 = 1$.

But if we only observe the system state $|S\rangle$, we will see that $|a(t)|^2 = e^{-(\Gamma)t}$. Therefore, the non Hermitian term does not mean that the total probability is really destroyed; It means that after ignoring the environment, the probability of the system part is lost.

However, in reality, , $(2\pi)^2(\Delta(m_m))|g(E_S)|^2\rho(E_S) = (m_e)(c)(a_0)$, that is,$(2\pi)(\Gamma)(G_N) = \frac{(h)}{(\Delta(m_m))}\frac{(e_0)}{(4\pi)(\varepsilon_0)}$.

If the system is completely in the system state at the beginning, that is,$a(0) = 1$, then the solution is: $a(t) = e^{-iE_St/(\hbar)}e^{-(\Gamma)t/2}$.

If the initial amplitude is not 1, then: $a(t) = a(0)e^{-iE_St/(\hbar)}e^{-(\Gamma)t/2}$, this solution has two parts. The first part: $e^{-iE_St/(\hbar)}$ is just the phase evolution in ordinary quantum mechanics. It is determined by the energy E_S and does not change the probability. Part 2: $e^{-(\Gamma)t/2}$, represents the exponential decrease of the probability amplitude of the system state over time.

And we know that probability is the square of the amplitude, so $P_S(t) = |a(t)|^2$. Substituting it into the solution yields $P_S(t) = |a(0)|^2e^{-(\Gamma)t}$.

If $a(0) = 1$, then : $P_S(t) = e^{-(\Gamma)t}$, which means that at time t, the probability of finding particles in the original system state still decreases exponentially. Correspondingly, the probability that the particle has entered the environment is $P_E(t) = 1 - e^{-(\Gamma)t}$. Therefore, the reduced probability in the system does not disappear, but transitions to the environmental state $|k\rangle$.

The probability of a complete closed system remains conserved: $|a(t)|^2 + \int dk\,|b_k(t)|^2 = 1$. That is to say, the probability originally in $|S\rangle$ did not disappear out of thin air, but transferred to the environmental part $\int dk\,|b_k(t)|^2 = 1 - e^{-(\Gamma)t}$. Therefore, replacing "environmental absorption system amplitude" with a more intuitive way is that after the system decays, the probability enters the decay products or environmental degrees of freedom.

Instead of the Hermitian term:$-i\frac{(\hbar)(\Gamma)}{2}$, it is a simplified description of the decay process when only looking at the "undegraded system state". It is equivalent to saying that after a short period of time, the system has a certain probability of leaving its original state and entering an environment or decay product that you are no longer tracking.

Therefore, the non Hermitian effective equation, $i(\hbar)\dot{a}(t) = \left[E_S - i\frac{(\hbar)(\Gamma)}{2}\right]a(t)$, is essentially a "decaying efficient Schr ö dinger equation".

So, it's simply amazing! Wonderful! Wonderful!

You see, the total Hamiltonian is:$H_{\text{tot}} = E_S|S\rangle\langle S| + \int dk\,E_k|k\rangle\langle k| + \int dk\,[g_k|S\rangle\langle k| + g_k^*|k\rangle\langle S|]$.

Part 1: The energy of the system itself, $E_S|S\rangle\langle S|$, where E_S is the energy of the system state $|S\rangle$. $|S\rangle\langle S|$ can be understood as "checking and retaining the part in the system state". Therefore, this statement states that if a quantum state is in the system, it has its own energy E_S.

Part 2: The energy of the environment itself, $\int dk\, E_k|k\rangle\langle k|$, there are many states in the environment $|k\rangle$, each state has its own energy E_k. Among them, k is a variable that marks the environment state; E_k is the energy corresponding to the state; $\int dk$ represents adding up all possible environmental states. The reason for using integration instead of ordinary summation is because the environmental energy is assumed to be continuously distributed.

Part Three: Interaction between System and Environment, $\int dk\ [g_k|S\rangle\langle k| + g_k^*|k\rangle\langle S|]$, this term is the most important because it allows quantum states to transfer between the system and the environment. Among them, $g_k|S\rangle\langle k|$ represents that the environmental state $|k\rangle$ can be transferred to the system state $|S\rangle$. And: $g_k^*|k\rangle\langle S|$ represents that the system state $|S\rangle$ can transition to the environment state $|k\rangle$. g_k is the strength of the interaction, indicating how strongly the system is coupled to the k environmental state. The superscript $*$ represents complex conjugation, which ensures that the total Hamiltonian satisfies the Hermitian property required by quantum mechanics, resulting in normal unitary evolution.

And now, according to our assumed equation, the relationship between interactions tells us that we don't need to consider so many things anymore. We will reduce the energy of the environment itself, the interaction between the system and the environment by ourselves. That is to say, we don't need to consider truncation and correction anymore. The real world will couple all the messy things by itself, and the residual of the linear approximation of the unitary evolution equation will eventually be a computable constant.

So for neutron decay, it is $(2\pi)^2(\Delta(m_m))|g(E_S)|^2\rho(E_S) = (m_e)(c)(a_0)$, or $(2\pi)(\Gamma)(G_N) = \frac{(h)}{(\Delta(m_m))}\frac{(e_0)}{(4\pi)(\varepsilon_0)}$.

2.3 The previously believed effective Schr ö dinger model

The neutron decay process is $n \to p + e^- + \bar{\nu}_e$.

1. Describing neutron decay using the ordinary non relativistic Schr ö dinger equation

The ordinary Schr ö dinger equation typically describes a non relativistic quantum system with a fixed number and type of particles: $i(\hbar)\frac{\partial}{\partial t}\Psi = H\Psi$, but in neutron decay, the initial state is a neutron and the final state contains proton, electron, and anti electron neutrinos. That is to say, the system transitions from one particle state to another state with a different number and type of particles. Therefore, the wave function cannot be written as a three-dimensional wave function with a fixed number of particles, but should be understood as a state in a multi particle Hilbert space: $|\Psi(t)\rangle = a_n(t)|n\rangle + \sum_f b_f(t)|f\rangle$, where the initial state is $|n\rangle$ and the final state can be written as $|f\rangle = |p, e^-, \bar{\nu}_e\rangle$. This form still satisfies an evolution equation similar to the Schr ö dinger equation, but the Hamiltonian must allow for the generation and annihilation of particles, thus essentially approaching quantum field theory.

We can separate the free Hamiltonian and weak interaction Hamiltonian:$H = H_0 + H_{\mathrm{W}}$, where H_0 describes the free motion of neutrons, protons, electrons, and neutrinos, and H_{W} describes transitions caused by weak interactions. On the basis of the initial and final states, the evolution equation can be approximated as $i(\hbar)\dot{a}_n(t) = E_n a_n(t) + \sum_f V_{nf} b_f(t)$, $i(\hbar)\dot{b}_f(t) = E_f b_f(t) + V_{fn} a_n(t)$, Among them, $V_{fn} = \langle f|H_{\mathrm{W}}|n\rangle$. At the beginning, it is assumed that the system must be in a neutron state: $a_n(0) = 1,\ b_f(0) = 0$, After eliminating the final state amplitude $b_f(t)$, the non local time equation for the initial state amplitude can be obtained:$\dot{a}_n(t) = -\frac{1}{(\hbar)^2}\sum_f |V_{fn}|^2 \int_0^t dt'\, e^{i(E_n - E_f)(t-t')/(\hbar)} a_n(t')$.

This is a common description of the 'equivalent Schr ö dinger equation'. It indicates that the amplitude of neutrons will decrease due to continuous coupling to the final state of decay. Under weak coupling, continuous spectrum, and Markov approximation, the right-hand side can be approximated as proportional to $a_n(t)$ at the current time:$\dot{a}_n(t) = -\left(\frac{(\Gamma)_n}{2(\hbar)} + \frac{i\Delta E}{(\hbar)}\right) a_n(t)$. So $a_n(t) = \exp\left(-\frac{(\Gamma)_n t}{2(\hbar)}\right)\exp\left(-\frac{i(E_n+\Delta E)t}{(\hbar)}\right)$.Therefore, the probability that the neutron is still in an undegraded state is $P_n(t) = |a_n(t)|^2 = \exp\left(-\frac{(\Gamma)_n t}{(\hbar)}\right)$. Define the average neutron decay time $(\tau)_n$ as$(\tau)_n = \frac{(\hbar)}{(\Gamma)_n}$, Then the usual exponential decay law is obtained: $P_n(t) = e^{-t/(\tau)_n}$, where $(\Gamma)_n$ is the decay width.

In the first-order perturbation approximation, the decay width is given by Fermi's Golden Rule: $(\Gamma)_n = \frac{(2\pi)}{(\hbar)}\sum_f |\langle f|H_{\mathrm{W}}|n\rangle|^2 \delta(E_f - E_n)$. For continuous final states, the sum needs to be replaced by phase space integration: $(\Gamma)_n = \frac{(2\pi)}{(\hbar)}\int d\Phi_3 |\mathcal{M}|^2 \delta(E_n - E_f)$, where $d\Phi_3$ is the three body phase space of proton, electron, and anti electron neutrinos, and $\mathcal{M}$ is the weak interaction matrix element.

This formula is not a simple spatial wave function calculation in ordinary potential energy scattering, but is obtained from the quantum field theory matrix elements of weak interactions. The standard theory of neutron decay is commonly referred to as Fermi type weak interaction theory or neutron beta decay effective field theory. NIST's review of neutron beta decay also describes it as a weak interaction process of fundamental charged current in baryons.

[NIST](https://www.nist.gov/publications/neutron-beta-decay)

2. Low energy Fermi effective Hamiltonian

In cases where the energy is much lower than the mass of the Wboson, weak interactions can be written as contact type four Fermi interactions. Its covariant form is roughly $\mathcal{H}_{\mathrm{W}} = \frac{G_F V_{ud}}{\sqrt{2}} \left[\bar{p}\gamma^{\mu}\left(g_V - g_A\gamma^5\right)n\right]\left[\bar{e}\gamma_{\mu}\left(1-\gamma^5\right)\nu_e\right] + \mathrm{h.c.}$, Among them, G_F is the Fermi constant; V_{ud} is the quark flavor mixing matrix element;g_V is vector coupling; g_A is axis vector coupling; γ^{μ} and γ^5 are Dirac matrices; h.c. represents Hermitian conjugation.

Neutrons and protons are not point particles, so the strong interaction structure will modify the coupling constant. Usually, $\lambda = \frac{g_A}{g_V}$ is used. Under allowed approximations, the structure of the total neutron decay rate can be summarized as $(\Gamma)_n \propto G_F^2 |V_{ud}|^2 \left(g_V^2 + 3g_A^2\right) f$, where f is a statistical rate function determined by the phase space of electrons and neutrinos. More accurate calculations also require radiation correction, recoil correction, nuclear structure correction, and so on. The modern standard model theory and effective field theory are precisely used to handle these high-precision corrections.

[OSTI](https://www.osti.gov/servlets/purl/2318474)

3. Non relativistic form: Fermi transition Hamiltonian

If we only care about the spin structure of low-energy neutrons and protons, we can conduct a non relativistic expansion on the nuclear part. The weak interaction Hamiltonian can be approximated as$H_{\mathrm{W}}^{\mathrm{NR}} \sim \frac{G_F V_{ud}}{\sqrt{2}} \int d^3x \left[g_V\, p^{\dagger} n\, e^{\dagger}\nu - g_A\, p^{\dagger}\boldsymbol{\sigma} n \cdot e^{\dagger}\boldsymbol{\sigma}\nu\right] + \mathrm{h.c.}$, The first term is Fermi type transition, and the second term is Gamow Taylor type transition. The spin quantity in the symbol is the two-dimensional Pauli spin quantity, and $\boldsymbol{\sigma}$ is the Pauli matrix. In this approximation, neutrons can be regarded as nucleons with two internal states: $|n\rangle$, $|p\rangle$, The weak interaction converts the internal state from $|n\rangle$ to $|p\rangle$, while producing electron and anti electron neutrinos.

Therefore, for the "nuclear space+electron space+neutrino space", it can indeed be written as a transition model similar to the Schr ö dinger equation:$i(\hbar)\frac{d}{dt}|\Psi(t)\rangle = \left(H_0 + H_{\mathrm{W}}^{\mathrm{NR}}\right)|\Psi(t)\rangle$, However, the computational results of this model still rely on quantum field processing of electrons and neutrinos. Treating electrons and neutrinos simply as ordinary single particle wavelet functions would result in the loss of key properties such as relativity, spin chirality, and particle generation.

4. The simplest two-level decay model

If you only want to understand the mathematical mechanism of decay, you can simplify neutron states and many final states into $H_{\mathrm{eff}} = \begin{pmatrix} E_n - \frac{i(\Gamma)_n}{2} & 0 \\ 0 & E_f \end{pmatrix}$

So the neutron component satisfies $i(\hbar)\frac{d}{dt}a_n(t) = \left(E_n - \frac{i(\Gamma)_n}{2}\right)a_n(t)$, The solution is $a_n(t) = e^{-iE_nt/(\hbar)}e^{-(\Gamma)_nt/(2\hbar)}$, and the non Hermitian term $-\frac{i(\Gamma)_n}{2}$ is not the true potential energy in the fundamental Hamiltonian, but an effective description obtained by eliminating all the decay final states that are not explicitly written. It represents the probability of flowing from the "neutron subspace" into the "proton+electron+anti electron neutrino" subspace.

More fully, all final states combine to form a continuous spectrum, rather than a single $|f\rangle$. It is the continuous spectrum that leads to stable exponential decay intervals and Fermi's golden rule.

The Schr ö dinger type effective model can well describe: 1. The decay of the initial neutron amplitude over time; 2. The relationship between neutron decay time and decay width; 3. Fermi's Golden Rule; 4. Electronic energy spectrum and final state phase space; 5. Relative contributions of different spin transitions; 6. Angle correlation and polarization correlation with appropriate modifications.

But it cannot independently and completely replace the Standard Model quantum field theory, for reasons including: 1. Electrons and neutrinos are relativistic particles; 2. Decay changes the number and type of particles; 3. Weak interactions have chiral structures; 4. Need to handle antiparticles; 5. High precision decay time calculation requires weak radiation correction; 6. The internal quark structure of protons and neutrons can affect matrix elements.

So the most accurate statement is: neutron decay can be described by an effective evolution equation of the Schr ö dinger type, but the decay matrix elements themselves must come from relativistic quantum field theory or low-energy weak interaction effective field theory.

The logical chain can be summarized as follows: standard model weak interaction $\longrightarrow$ Fermi effective Hamiltonian $\longrightarrow$ initial continuous final state model $\longrightarrow$ Schr ö dinger type effective equation $\longrightarrow$ $P_n(t) = e^{-t/(\tau)_n}$.

Fact: True neutron decay is not a strictly two-level system, as the decaying protons, electrons, and antineutrinos have countless different combinations of momentum and energy. The two-level model can only be used to understand why the probability of neutron states decreases, and cannot accurately calculate the true neutron decay time.

The Schr ö dinger equation is used to describe the time evolution of decay, usually using the effective Hamiltonian of "discrete initial state+continuous final state" to obtain exponential decay, decay width, and Fermi's golden rule.

Neutron decay is a process of particle species change caused by weak interactions, resulting in the production of electron and anti electron neutrinos. Strict theory requires quantum field theory. The commonly used theories in practice are relativistic quantum field theory or its low-energy effective theory.

3 The meaning of neutron decay time as a 'constant'

Saying 'neutron decay time is a constant' usually does not mean that every neutron can decay at exactly the same time, but rather that the decay of free neutrons follows a stable statistical pattern. We cannot accurately predict when a single neutron will decay; It may decay quickly or survive for a long time. The specific decay time has randomness.

In physics, the term 'average decay time' is commonly used to describe the decay characteristics of free neutrons, denoted as τ_n, with a value of approximately 880 s, which is approximately 15minutes. The meaning of average decay time is that if a sufficient number of neutrons are observed and their time from generation to decay is statistically averaged, the result will gradually approach this value. It is not the actual decay time of every neutron.

The proportion of free neutrons that have not decayed after a time of t can be expressed exponentially as $P(t) = e^{-t/\tau_n}$. When time equals the average decay time, the proportion that has not decayed is approximately e^{-1}, which is around 36.8%; The proportion that has already decayed is approximately

63.2%. This indicates that the average decay time reflects the overall statistical characteristics, rather than the determined fate of individual particles.

The decay constant λ_n corresponds to the average decay time, and the relationship between the two is $\lambda_n = 1/\tau_n$. The decay constant represents the probability of a neutron that has not yet decayed occurring in a unit of time. As long as the environmental conditions remain unchanged, this probability rate remains constant. Therefore, how long a neutron has survived does not change its basic probability of decay in the next moment.

So, the true meaning of "constant" is that the microscopic decay law and statistical parameters of free neutrons remain stable, rather than the decay time being fixed in advance. It should be noted that this conclusion mainly applies to free neutrons; When neutrons are inside atomic nuclei, their nuclear structure and energy conditions may alter their decay behavior.

The "special feature" of "neutron decay time is a constant" mainly refers to its physical constant that describes the velocity of quantum random processes, unlike length and mass, which are just ordinary static properties of an object.

More precisely, the average decay time of free neutrons is denoted as τ_n, which represents that the statistical average decay time of a large number of free neutrons under the same conditions tends to approach the same stable value. It doesn't mean that every neutron will decay exactly τ_n seconds later.

This constant is special in that it has the following meanings simultaneously:

Firstly, it is a statistical constant. When a single neutron decays cannot be determined, only probability can be given; But the overall decay law of a large number of neutrons is very stable. The survival probability satisfies $P_{\text{survive}}(t) = e^{-t/\tau_n}$. Therefore, individual behavior is random, while overall patterns are precise.

Secondly, it is a constant of memoryless processes. How long a neutron has existed does not change its basic probability of decay in the next short period of time. The decay constant is $\lambda_n = 1/\tau_n$, which remains constant when the environment remains unchanged. This is different from the notion that people or machines are more prone to damage as they age; Neutrons do not undergo aging in the usual sense.

Thirdly, it is the result of fundamental interactions. The neutron decay time is not a artificially

determined number, but an approximate equivalent result of the coupling of weak interaction strength, quark mixing, internal structure of nucleons, and available energy of decay final states to the fundamental interaction. The process that can occur with free neutrons is $n \to p + e^- + \bar{\nu}_e$. So, neutron decay time condenses the information of microscopic weak interactions into a measurable number.

Fourthly, it has universality and conditions. Under the same physical conditions, any free neutron should follow the same average decay time; But if neutrons are confined within the atomic nucleus, the nuclear binding energy and structure will change whether decay is allowed and its rate, and the decay time of free neutrons cannot be directly applied at this time.

Therefore, the meaning of "constant" is: it is a constant at the intersection of quantum randomness, interaction, and stable statistical laws.

4 References

Some of the deductions and theories in this article are already based on professional knowledge, while the other assumptions and connections are entirely based on my personal research, so there are no references in this article.

In addition, I would like to express my sincere gratitude to artificial intelligence. Although it can be stubborn like a paratrooper at times, it is still a great personal assistant.

一个中子为什么会衰变的新的可能性解释

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摘要：如果中子衰变时间是一个“常数”呢，我们给出一个新的近似等式，然后在这个假设下尝试理解它。

关键词：中子衰变, 中子质量, 质子质量，相互作用

5 假设中子衰变时间是一个“常数”

假设中子衰变时间：$(\tau) = (2\pi)^3 \dfrac{(2)(a_0)(R_\infty)(\Delta(m_m))}{(h)} = (2\pi)\dfrac{[\alpha_0](\Delta(m_m))}{(\hbar)}$。

5.1 一种可能得到中子衰变时间的计算方法

为了让改过的薛定谔方程产生这个衰变时间公式：$(\tau) = (2\pi)^3 \dfrac{(2)(a_0)(R_\infty)(\Delta(m_m))}{(h)} = (2\pi)\dfrac{[\alpha_0](\Delta(m_m))}{(\hbar)}$。从薛定谔方程本身出发，我们可以引入一个“有效薛定谔方程”：$i(\hbar)\dfrac{\partial}{\partial t}\psi = \left[-\dfrac{(\hbar)^2}{2(m_e)}\nabla^2 + V - i\dfrac{(\hbar)}{2(\tau)}\right]\psi$，其中 V 可以任意，但我们也可以从拉格朗日量推导出有效包含复势的薛定谔方程，通过一个“等效原理”：将耗散函数引入变分得到运动方程，然后在慢变近似下映射到薛定谔方程。

我们还可以考虑“无反射势”或“复势”的具体形式，比如取 $V = 0$，则方程变为 $i(\hbar)\partial_t\psi = -\dfrac{(\hbar)^2}{2(m_e)}\nabla^2\psi - i\dfrac{(\hbar)}{2(\tau)}\psi$。这个方程的解是 $\psi \sim e^{-iEt/(\hbar)}e^{-t/(2(\tau))}$，归一化到体积 V，则概率衰变时间为 (τ)。将 (τ) 代入，虚部就是 $-\dfrac{i}{2}(\hbar)\dfrac{(\hbar)}{(2\pi)[\alpha_0](\Delta(m_m))}$。

乱七八糟的把这些整理一下，最后的顺序就是，在薛定谔方程中引入一个复势虚部，其大小由该 (τ) 反推得到。具体做法如下。

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1. 原始（无耗散）薛定谔方程：$i(\hbar)\frac{\partial\psi}{\partial t}=\left[-\frac{(\hbar)^2}{2(m_e)}\nabla^2+V(\mathbf{r})\right]\psi$。

2. 修改为复势薛定谔方程，引入一个常数复势：$V_{\text{eff}}(\mathbf{r})=V(\mathbf{r})-i\frac{(\hbar)}{2(\tau)}$，则方程变为：$i(\hbar)\frac{\partial\psi}{\partial t}=\left[-\frac{(\hbar)^2}{2(m_e)}\nabla^2+V(\mathbf{r})-i\frac{(\hbar)}{2(\tau)}\right]\psi$。

3. 代入目标衰变时间公式，将 $(\tau)=(2\pi)\frac{[\alpha_0](\Delta(m_m))}{(\hbar)}$ 代入虚部：$\frac{(\hbar)}{2(\tau)}=\frac{(\hbar)}{2}\cdot\frac{(\hbar)}{(2\pi)[\alpha_0](\Delta(m_m))}$，因此复势可显式写为：$V_{\text{eff}}(\mathbf{r})=V(\mathbf{r})-i\,\frac{(\hbar)^2}{(4\pi)[\alpha_0](\Delta(m_m))}$，薛定谔方程成为：$i(\hbar)\frac{\partial\psi}{\partial t}=\left[-\frac{(\hbar)^2}{2(m_e)}\nabla^2+V(\mathbf{r})-i\,\frac{(\hbar)^2}{(4\pi)[\alpha_0](\Delta(m_m))}\right]\psi$。

4. 证明衰变时间一致，若 $V(\mathbf{r})$ 与时间无关，则方程的解具有形式：$\psi(\mathbf{r},t)=u(\mathbf{r})\,e^{-iEt/(\hbar)}\,e^{-t/(2(\tau))}$，其中 $u(\mathbf{r})$ 满足定态方程：$\left[-\frac{(\hbar)^2}{2(m_e)}\nabla^2+V(\mathbf{r})\right]u(\mathbf{r})=E\,u(\mathbf{r})$，因此概率密度随时间变化为：$|\psi(\mathbf{r},t)|^2=|u(\mathbf{r})|^2\,e^{-t/(\tau)}$，从而衰变时间正是：$(\tau)=(2\pi)\frac{[\alpha_0](\Delta(m_m))}{(\hbar)}$。

6 中子衰变时间的计算方法是怎么得到的

既然按照这样的推导逻辑没问题，那么，中子衰变时间的这个结果是怎么来的呢，当然，这个也是先有猜测，再有结果。即，我认为质子或者质子质量是个稳定结构，中子或者中子质量是个完美结构，那么它们的某种差或者商值是不是也是某种固定的呢，然后经过我的胡思乱想，发现，或者说，认为中子和质子的质量差 $(\Delta(m_m))$、中子衰变时间 (τ) 用物理常数符号代表的数值积能表示出来。

即，$(m_n)-(m_p)=(\Delta(m_m))\approx\frac{(m_e)}{(4\pi)}\times\frac{1}{(4\pi)(\varepsilon_0)(c)}$，$2.5(m_e)\approx2.3(m_e)$。

即，$(\tau)\approx(\frac{1}{(4\pi)(\varepsilon_0)(c)})^2$，$880\approx900$。

然后我们简单粗暴地发现，不考虑物理常数符号后面代表的量纲，并且认为物理常数符号后面代表的量纲和数值分开看的话，现在，仅看数值的话，忽略近似符号，中子衰变时间有几种等式。

1. 即，$(\tau)=(4\pi)^2\frac{(\Delta(m_m))^2}{(m_e)^2}=(2\pi)^2\frac{(\Delta(m_m))^2(c)^4}{(m_n)}$。
2. 即，$(\tau)=(2\pi)^3\frac{(2)(a_0)(\Delta(m_m))}{(m_e)(G_N)}=(2\pi)^3\frac{(2)(a_0)(R_\infty)(\Delta(m_m))}{(h)}$。
3. 即，$(\tau)=(2\pi)\frac{(\Delta(m_m))}{(m_e)(a_0)(c)}=(2\pi)^2\frac{(\Delta(m_m))}{(m_e)(2\pi)(a_0)(c)}$。
4. 即，$(\tau)=(2\pi)\frac{(\Delta(m_m))}{(m_e)(R_\infty)\frac{(e_0)}{(4\pi)(\varepsilon_0)}}=(2\pi)\frac{(G_N)(\Delta(m_m))}{(h)\frac{(e_0)}{(4\pi)(\varepsilon_0)}}$。
5. 即，$\frac{(2\pi)}{(c)(\tau)}(G_N)=\frac{(h)}{(\Delta(m_m))(c)}\frac{(e_0)}{(4\pi)(\varepsilon_0)}$。
6. 即，$\frac{(2\pi)^2(r)^2}{(c)(\tau)}=\frac{(h)}{(\Delta(m_m))[\alpha_0](c)}$。

一个很有趣的想法是，我本来打算写六种新的可能的中子衰变时间的计算方法的，但是写到中途我发

现一件事情，即，用改过的薛定谔方程非常有美感啊，或者说，感觉非常自然啊，就像用这里面的唯一有平方的等式去套拉格朗日方程得到的东西一样，看起来非常自然。

那么，这里面会有什么内在的联系吗，然后我做了一件非常有趣的事情，就是给出来这个可能的联系。

当然，如果有个天才，在算康普顿散射时，突发奇想，把中子和质子的质量差类比为摆线公式中的圆或圆的轨迹，那么他会想到当速度处于特殊数值时我们算出的这个波长有什么特别的地方吗，假如这个天才意识到这个波长有特别的地方并且把它和中子衰变或中子衰变时间联系到一起，那么他就有可能得到$\frac{(2\pi)^2(r)^2}{(c)(\tau)} = \frac{(h)}{(\Delta(m_m))[\alpha_0](c)}$。但是现实世界给的迷惑项太多了，终究还是没人能想到这个东西。

已知，现在这个改过的薛定谔方程是非幺正的，那么，如果它成立，能够得到它的封闭系统的幺正演化方程应该是怎样的，再简单点儿，能够得到它的一个最简单的封闭系统的幺正演化方程应该是怎样的，有趣的是，如果再牵扯的复数，那么，能够得到它的一个最简单的复空间封闭系统的幺正演化方程应该是怎样的，如果你有一定的数学物理基础，那么最简单的办法就是问问人工智能呗。

有趣的是，如果用封闭系统的幺正演化方程得到的近似解，我们会得到衰减率 $(\Gamma) = \frac{1}{(\tau)} = (2\pi)|\Omega|^2\rho(E_0)$，如果没发现有什么特别之处的同学，可能会说，这又是一个平平无奇的假设。

但是，如果要最后能得到我们这个改过的薛定谔方程的解，那么条件就得是 $(\tau) = (2\pi)^3\frac{(2)(a_0)(R_\infty)(\Delta(m_m))}{(h)} = (2\pi)^2\frac{[\alpha_0](\Delta(m_m))}{(h)} = (2\pi)\frac{[\alpha_0](\Delta(m_m))}{(\hbar)}$。

也就是，$|\Omega|^2\rho(E_0) = \frac{(\hbar)}{(2\pi)^2[\alpha_0](\Delta(m_m))} = \frac{h}{(2\pi)^3[\alpha_0](\Delta(m_m))}$。

即，$(2\pi)^3|\Omega|^2\rho(E_0) = \frac{h}{[\alpha_0](\Delta(m_m))}$。

即，$(2\pi)^2(\Delta(m_m))|\Omega|^2\rho(E_0) = (m_e)(c)(a_0)$。

然后我们把量纲还原的就是，$(2\pi)^2(r)^2(\Delta(m_m))|\Omega|^2\rho(E_0) = (m_e)(c)(a_0)$。

然后，忽略量纲，再考虑一个有趣的地方，即，$(2\pi)(\Gamma)(G_N) = \frac{(h)}{((\Delta(m_m))}\frac{(e_0)}{(4\pi)(\varepsilon_0)}$。

当我把这个等式写出来并且放在中子衰变的推理过程中时，我就觉得这可能是对的，因为这个等式至少告诉我这个等式提供了一个衰变环境。

那么，这个中子衰变时间的计算方法是怎么来的，答案是猜出来的，那么，它合理吗，答案是至少现在看这个我给的过程不合理，但是，当它处理到和相互作用有关系时，你还没有一点点儿触动，我觉得，当真正的答案放在你面前时，但这个答案和你已知的且认为是部分正确的答案冲突时，我觉得你也分不出来这个真正的答案到底是不是对的。

6.1 符合中子衰变时间的一个计算方程

那么，现在，回到我们故事的出发点，$(m_n)-(m_p)=(\Delta(m_m))$，如果我们认为中子是个完美结构，质子是个稳定结构，那么它们之间的差就是可以计算的，而在数学上，“封闭系统，幺正演化”也是一个可以给出推导过程的东西。也就是说，现在把中子衰变看成是一个最简的两级模型，可以把它先想成一个非常简化的“中子态—衰变后态”模型。

然后，事实是真正的中子衰变不是严格的两能级系统，因为衰变后的质子、电子和反中微子有无数种不同动量和能量组合。两能级模型只能用来理解“中子态的概率为什么会减少”，不能精确计算真实中子衰变时间。

但是，现在按照这个理解，中子衰变就是严格的两级系统，这个两级模型可以用来理解“中子态的概率为什么会减少”，也可以精确计算真实中子衰变时间。当然，这个时候你就要理解你要计算的真实中子衰变时间到底是什么了。

那么，现在我们逆向推理一下，如果我们假设 $(\tau)=(2\pi)\dfrac{[\alpha_0](\Delta(m_m))}{(\hbar)}=\dfrac{1}{(\Gamma)}$，那么从最简单的地方考虑，方程可以是，$i(\hbar)\dfrac{\partial\psi}{\partial t}=\left[E_S-i\,\dfrac{(\hbar)(\Gamma)}{2}\right]\psi$，也就是上面我们给的方程，但是，现在，这是一个非幺正非封闭的系统。

然后，如果我们再次逆向，我们希望得到一个封闭的幺正演化方程，在消去环境后可以得到原来的方程。而我们知道一个最简单的严格封闭幺正模型是二维复空间模型：$\mathbb{C}^2$；一个最简单的能近似给出不可逆指数衰减的封闭幺正模型是“一个离散系统态加一个连续谱环境”：$\mathbb{C}\oplus L^2(\mathbb{R})$。

那么，现在，这个最简单的封闭模型就可能是 $\mathcal{H}_{\rm tot}=\mathbb{C}\oplus L^2(\mathbb{R})$，也就是“一个系统态”加上“一个具有连续能谱的环境”。它的总哈密顿量可写成：$H_{\rm tot}=E_S|S\rangle\langle S|+\int dk\,E_k|k\rangle\langle k|+\int dk\,\left[g_k|S\rangle\langle k|+g_k^*|k\rangle\langle S|\right]$，总态可以写成：$|\Psi(t)\rangle=a(t)|S\rangle+\int dk\,b_k(t)|k\rangle$，其幺正演化方程为：$i(\hbar)\dfrac{d}{dt}|\Psi(t)\rangle=H_{\rm tot}|\Psi(t)\rangle$。

在连续谱、弱耦合和 Markov 近似下，系统振幅满足：$i(\hbar)\dot{a}(t)=\left[E_S-i\dfrac{(\hbar)(\Gamma)}{2}\right]a(t)$。

即，$a(t)=a(0)e^{-iE_St/(\hbar)}e^{-(\Gamma)t/2}$，$|a(t)|^2=|a(0)|^2e^{-(\Gamma)t}$。

6.2 一个“完整封闭、严格幺正”的有效方程

好了，掺杂了很多不明觉厉的东西，现在我们仔细来说一下这个东西。

上面这段话是在描述一个很简单的“量子系统 + 环境”模型。核心意思是：一个量子态可以先待在系统里，也可以通过相互作用跑到环境中；环境有很多可能的能量状态。

若环境只有一个态，或者只有有限多个态，振幅会在系统和环境之间来回交换，通常产生振荡与回流。但是环境具有连续谱时，系统泄漏出去的振幅会分散到大量不同能量的环境态中。

每一个环境态具有不同相位：$e^{-iE_k(t-t')/(\hbar)}$，时间变长后，不同能量对应的相位彼此错开、相互抵消。泄漏出去的振幅很难重新整齐地回到系统态中。因此，从系统的角度看，概率像是“单向流入”环境。

但是，如果连续环境吸收了系统的振幅，在弱耦合和无记忆近似下，这种流失成为恒定速率的指数衰减，于是完整厄米哈密顿量的约化动力学表现为带有虚部的非厄米有效哈密顿量。而虚部不是凭空加上的。它来自系统态不断把概率振幅传入连续环境态，所以，当我们不再追问环境的细节时，这种流出表现为衰减，所以，当我们只看系统部分时，会得到带虚部的有效方程。

所以，首先我们假设总态为：$|\Psi(t)\rangle = a(t)|S\rangle + \int dk\, b_k(t)|k\rangle$，其中，$a(t)$ 是系统态 $|S\rangle$ 的概率振幅；$b_k(t)$ 是环境中第 k 个连续态 $|k\rangle$ 的概率振幅；一开始如果粒子完全在系统中，则：$a(0)=1$，$b_k(0)=0$。

完整哈密顿量为：$H_{\text{tot}} = E_S|S\rangle\langle S| + \int dk\, E_k|k\rangle\langle k| + \int dk\, [g_k|S\rangle\langle k| + g_k^*|k\rangle\langle S|]$，其中：$E_S$ 是系统态能量；E_k 是环境态能量；g_k 是系统和环境态之间的耦合强度。总演化严格遵循：$i(\hbar)\dfrac{d}{dt}|\Psi(t)\rangle = H_{\text{tot}}|\Psi(t)\rangle$，且总概率守恒：$|a(t)|^2 + \int dk\, |b_k(t)|^2 = 1$。

现在把总态代入薛定谔方程，并分别比较系统态和环境态前面的系数，得到：$i(\hbar)\dot{a}(t) = E_S a(t) + \int dk\, g_k b_k(t)$ 以及 $i(\hbar)\dot{b}_k(t) = E_k b_k(t) + g_k^* a(t)$。其中第一个等式表示：系统振幅不仅按自己的能量演化，也受到环境反馈。第二个等式表示：环境中的每一个态都由系统态不断“灌入”振幅。

也就是说，第二个等式可以解出环境振幅，那么先处理第二个方程：$i(\hbar)\dot{b}_k(t) = E_k b_k(t) + g_k^* a(t)$，在初始条件 $b_k(0)=0$ 下，它的解为：$b_k(t) = -\dfrac{i}{(\hbar)} g_k^* \int_0^t dt'\, e^{-iE_k(t-t')/(\hbar)} a(t')$。这句话的含义是：在时刻 t' 时，系统向环境态 $|k\rangle$ 转移了一点振幅；这点振幅之后按环境自身的能量 E_k 演化，最后共同构成时刻 t 的环境振幅。

然后把这个环境振幅的解代回系统方程，也就是把上面的 $b_k(t)$ 代入第一个等式：$i(\hbar)\dot{a}(t) = E_S a(t) + \int dk\, g_k b_k(t)$，得到：$i(\hbar)\dot{a}(t) = E_S a(t) - \dfrac{i}{(\hbar)} \int_0^t dt' \int dk\, |g_k|^2 e^{-iE_k(t-t')/(\hbar)} a(t')$，也可写为：$\dot{a}(t) = -\dfrac{iE_S}{(\hbar)} a(t) - \int_0^t dt'\, K(t-t')a(t')$，其中记忆核为：$K((\tau)) = \dfrac{1}{(\hbar)^2} \int dk\, |g_k|^2 e^{-iE_k(\tau)/(\hbar)}$，这时还没有做任何近似，完整系统仍然是幺正的，但是你会看到，系统现在依赖过去所有时刻的 $a(t')$。这称为“记忆效应”。

现在我们假设，或者说，我们有一个 Markov 近似，Markov 近似的核心假设是：环境的关联时间很短，而系统振幅变化较慢。因此，在记忆核显著不为零的短时间区间内，可以近似：$a(t') \approx a(t)$，并且把原先的积分上限从 t 延长到无穷远：$\int_0^t dt' \longrightarrow \int_0^\infty dt'$，这样，系统就不再依赖详细的过去历史，只依赖

当前的 $a(t)$。这就是“无记忆”近似。

为了更方便地看出结果，定义频率：$\omega_S = \frac{E_S}{(\hbar)}$，$\omega_k = \frac{E_k}{(\hbar)}$，并写：$a(t) = c(t)e^{-i\omega_S t}$，在弱耦合条件下，$c(t)$ 随时间变化很慢。经过 Markov 近似后，得到：$\dot{c}(t) = -\left(\frac{(\Gamma)}{2} + i\Delta\omega\right)c(t)$，其中，$(\Gamma) = \frac{(2\pi)}{(\hbar)}\int dk\,|g_k|^2\delta(E_S - E_k)$，而 $\Delta\omega$ 是环境带来的能量修正，也称为能级位移或 Lamb shift 型修正。其中，环境的态密度若定义为：$\rho(E) = \int dk\,\delta(E - E_k)$，并且在 E_S 附近把耦合近似视为常数，则：$(\Gamma) = \frac{(2\pi)}{(\hbar)}|g(E_S)|^2\rho(E_S)$，这就是费米黄金规则。

同时，这也是我们上面提到的衰减率 $(\Gamma) = \frac{1}{(\tau)} = (2\pi)|\Omega|^2\rho(E_0)$，如果环境通道有连续谱，并且初始时刻 ϕ 处于“空”态，那么在 Wigner–Weisskopf 近似下，将 ϕ 自由度求迹后，ψ 满足：$i(\hbar)\frac{\partial\psi}{\partial t} = \left[E_S - i\,\frac{(\hbar)(\Gamma)}{2}\right]\psi$，其中衰减率为：$(\Gamma) = (2\pi)|\Omega|^2\rho(E_0)$，$|\Omega|^2$ 是描述系统与环境之间的跃迁耦合强度，即相互作用“有多强”，而 $\rho(E_0)$ 就是描述环境在能量 E_0 附近的态密度，即可供粒子跃迁到达的末态“有多少个通道”。

现在我们将相位因子乘回去，得到：$a(t) = a(0)\exp\left[-\frac{i}{(\hbar)}(E_S + \Delta E)t\right]\exp\left[-\frac{(\Gamma)t}{2}\right]$，其中，$\Delta E = \hbar\Delta\omega$，如果暂时忽略能量修正 ΔE，那么，$a(t) = a(0)e^{-iE_S t/(\hbar)}e^{-(\Gamma)t/2}$，于是，$|a(t)|^2 = |a(0)|^2 e^{-(\Gamma)t}$。所以，振幅以速率 $(\Gamma)/2$ 衰减；概率以速率 (Γ) 衰减；概率衰变时间为：$(\tau) = \frac{1}{(\Gamma)}$。

现在这个解等价于这个有效方程：$i(\hbar)\dot{a}(t) = \left[E_S + \Delta E - i\frac{(\hbar)(\Gamma)}{2}\right]a(t)$，其中虚部：$-i\frac{(\hbar)(\Gamma)}{2}$，意味着系统态的概率流向环境。它并不意味着总概率消失。完整系统仍满足：$|a(t)|^2 + \int dk\,|b_k(t)|^2 = 1$。非厄米有效方程只是在说：你只观察系统态、忽略环境态后，系统部分不再是封闭的。

这也是上面我们说的，当我们不再追问环境的细节时，这种流出表现为衰减，所以，当我们只看系统部分时，会得到带虚部的有效方程。

而现在我们假设有效虚势为：$-i\frac{(\hbar)^2}{(4\pi)[\alpha_0](\Delta(m_m))}$，而一般有效方程的虚势为：$-i\frac{(\hbar)(\Gamma)}{2}$，比较二者后发现，$\frac{(\hbar)(\Gamma)}{2} = \frac{(\hbar)^2}{(4\pi)[\alpha_0](\Delta(m_m))}$，因此，$(\Gamma) = \frac{(\hbar)}{(2\pi)[\alpha_0](\Delta(m_m))}$。

而系统振幅就是 $a(t) = a(0)e^{-iE_S t/(\hbar)}\exp\left[-\frac{(\hbar)t}{(4\pi)[\alpha_0](\Delta(m_m))}\right]$，系统仍存留的概率是 $P_S(t) = |a(t)|^2 = |a(0)|^2\exp\left[-\frac{(\hbar)t}{(2\pi)[\alpha_0](\Delta(m_m))}\right]$，相应衰变时间为，$(\tau) = \frac{1}{(\Gamma)} = \frac{(2\pi)[\alpha_0](\Delta(m_m))}{(\hbar)}$。

所以，这个特定解实际上说的是，系统态具有能量 E_S，同时以衰减率 (Γ) 向连续环境通道泄漏；系统振幅按 $e^{-(\Gamma)t/2}$ 衰减，而系统中的概率按 $e^{-(\Gamma)t}$ 衰减。

在完整封闭系统中：$|\Psi(t)\rangle = a(t)|S\rangle + \int dk\,b_k(t)|k\rangle$，总概率始终守恒：$|a(t)|^2 + \int dk\,|b_k(t)|^2 = 1$。

但只观察系统态 $|S\rangle$ 的话，会看到：$|a(t)|^2 = e^{-(\Gamma)t}$，因此，非厄米项并不是说总概率真的被毁灭；它

表示你忽略环境后，系统部分的概率流失了。

但是，实际上，$(2\pi)^2(\Delta(m_m))|g(E_S)|^2\rho(E_S) = (m_e)(c)(a_0)$，即，$(2\pi)(\Gamma)(G_N) = \dfrac{(h)}{(\Delta(m_m))}\dfrac{(e_0)}{(4\pi)(\varepsilon_0)}$。

如果初始时系统完全处于系统态，即：$a(0) = 1$，则解为：$a(t) = e^{-iE_St/(\hbar)}e^{-(\Gamma)t/2}$。

如果初始振幅不是 1，则：$a(t) = a(0)e^{-iE_St/(\hbar)}e^{-(\Gamma)t/2}$，这个解有两个部分。第一部分：$e^{-iE_St/(\hbar)}$，只是普通量子力学中的相位演化。它由能量 E_S 决定，不会改变概率。第二部分：$e^{-(\Gamma)t/2}$，表示系统态的概率振幅随时间指数减小。

而我们知道概率是振幅的模平方，因此：$P_S(t) = |a(t)|^2$，代入解可得：$P_S(t) = |a(0)|^2e^{-(\Gamma)t}$。

若 $a(0) = 1$，则：$P_S(t) = e^{-(\Gamma)t}$，这说明在时间 t 时，仍然发现粒子处于原系统态中的概率，按指数规律减少。相应地，粒子已经进入环境的概率为：$P_E(t) = 1 - e^{-(\Gamma)t}$，因此，系统中减少的概率并没有消失，而是转移到了环境态 $|k\rangle$ 中。

完整封闭系统的概率仍守恒：$|a(t)|^2 + \int dk\,|b_k(t)|^2 = 1$。也就是说，原来在 $|S\rangle$ 中的概率并没有凭空消失，而是转移到环境部分 $\int dk\,|b_k(t)|^2 = 1 - e^{-(\Gamma)t}$，所以，把“环境吸收系统振幅”换成更直观的话就是，系统衰变后，概率进入了衰变产物或环境自由度中。

而非厄米项：$-i\dfrac{(\hbar)(\Gamma)}{2}$，就是只看“未衰变系统态”时，对衰变过程的简化描述。它等价于说，每经过一小段时间，系统都有一部分概率离开原状态，进入你不再追踪的环境或衰变产物。

因此非厄米有效方程，$i(\hbar)\dot{a}(t) = \left[E_S - i\dfrac{(\hbar)(\Gamma)}{2}\right]a(t)$，本质上就是一个“衰变态的有效薛定谔方程”。

所以，简直就是，妙！妙！妙！

你看，总哈密顿量是：$H_{\text{tot}} = E_S|S\rangle\langle S| + \int dk\,E_k|k\rangle\langle k| + \int dk\,[g_k|S\rangle\langle k| + g_k^*|k\rangle\langle S|]$。

第一部分：系统自己的能量，$E_S|S\rangle\langle S|$，这里 E_S 是系统态 $|S\rangle$ 的能量。$|S\rangle\langle S|$ 可以理解为“检查并保留在系统态中的部分”。因此这一项表示：如果量子态在系统里，它自身具有能量 E_S。

第二部分：环境自己的能量，$\int dk\,E_k|k\rangle\langle k|$，环境中有很多状态 $|k\rangle$，每个状态有自己的能量 E_k。其中，k 是标记环境状态的变量；E_k 是对应状态的能量；$\int dk$ 表示把所有可能的环境状态都加起来。而之所以使用积分而不是普通求和，是因为环境能量被假设为连续分布。

第三部分：系统和环境之间的相互作用，$\int dk\,[g_k|S\rangle\langle k| + g_k^*|k\rangle\langle S|]$，这一项最重要，因为它允许量子态在系统和环境之间转移。其中：$g_k|S\rangle\langle k|$ 表示环境状态 $|k\rangle$ 可以转移到系统态 $|S\rangle$。而：$g_k^*|k\rangle\langle S|$ 表示系统态 $|S\rangle$ 可以转移到环境状态 $|k\rangle$。g_k 是相互作用强度，表示系统和第 k 个环境状态耦合得有多强。上标 $*$ 表示复共轭，它保证总哈密顿量满足量子力学要求的厄米性，从而产生正常的幺正演化。

而现在按照我们假设的等式，相互作用之间的关系告诉我们，不用再考虑那么多东西了，环境自己的能量、系统和环境之间的相互作用我们会自己约化的，也就是说，不用再考虑什么截断和修正，现实世界会自己把乱七八糟的东西统统耦合掉，最终幺正演化方程的线性近似的残留量会是一个可计算的常数。

那么对于中子衰变来说，就是 $(2\pi)^2(\Delta(m_m))|g(E_S)|^2\rho(E_S) = (m_e)(c)(a_0)$，或者，$(2\pi)(\Gamma)(G_N) = \frac{(h)}{(\Delta(m_m))}\frac{(e_0)}{(4\pi)(\varepsilon_0)}$。

6.3 之前认为的有效的薛定谔模型

中子衰变过程是 $n \to p + e^- + \bar{\nu}_e$。

1. 用普通非相对论薛定谔方程描述中子衰变

普通薛定谔方程通常描述固定粒子数、固定粒子种类的非相对论量子系统：$i(\hbar)\frac{\partial}{\partial t}\Psi = H\Psi.$，但中子衰变中，初态是一个中子，末态包含质子、电子和反电子中微子。也就是说，系统从一个粒子态跃迁到了另一个粒子数和粒子种类不同的态。因此，波函数不能只写成一个固定粒子数的三维波函数，而应当理解为多粒子希尔伯特空间中的态：$|\Psi(t)\rangle = a_n(t)|n\rangle + \sum_f b_f(t)|f\rangle$，其中初态为 $|n\rangle$，末态可以写成 $|f\rangle = |p, e^-, \bar{\nu}_e\rangle$。这种形式仍然满足类似薛定谔方程的演化方程，但哈密顿量必须允许粒子的产生和湮灭，因此本质上已经接近量子场论。

我们可以把自由哈密顿量和弱相互作用哈密顿量分开：$H = H_0 + H_{\mathrm{W}}$，其中，H_0 描述中子、质子、电子和中微子的自由运动，H_{W} 描述弱相互作用引起的跃迁。在初态和末态基底上，演化方程可以近似写成 $i(\hbar)\dot{a}_n(t) = E_n a_n(t) + \sum_f V_{nf} b_f(t)$，$i(\hbar)\dot{b}_f(t) = E_f b_f(t) + V_{fn} a_n(t)$，其中 $V_{fn} = \langle f|H_{\mathrm{W}}|n\rangle$。初始时假设系统一定处于中子态：$a_n(0) = 1,\ b_f(0) = 0$，消去末态振幅 $b_f(t)$ 后，可以得到初态振幅的非局域时间方程：$\dot{a}_n(t) = -\frac{1}{(\hbar)^2}\sum_f |V_{fn}|^2 \int_0^t dt'\, e^{i(E_n - E_f)(t-t')/(\hbar)} a_n(t')$。

这就是一种常见的“等价薛定谔方程”描述。它说明中子振幅会因为不断耦合到连续的衰变末态而减少。在弱耦合、连续谱和马尔可夫近似下，可以把右侧近似为与当前时刻的 $a_n(t)$ 成正比：$\dot{a}_n(t) = -\left(\frac{(\Gamma)_n}{2(\hbar)} + \frac{i\Delta E}{(\hbar)}\right) a_n(t)$。于是 $a_n(t) = \exp\left(-\frac{(\Gamma)_n t}{2(\hbar)}\right)\exp\left(-\frac{i(E_n+\Delta E)t}{(\hbar)}\right)$。因此中子仍处于未衰变态的概率为 $P_n(t) = |a_n(t)|^2 = \exp\left(-\frac{(\Gamma)_n t}{(\hbar)}\right)$。定义中子平均衰变时间 $(\tau)_n$ 为 $(\tau)_n = \frac{(\hbar)}{(\Gamma)_n}$，则得到通常的指数衰变律：$P_n(t) = e^{-t/(\tau)_n}$，这里的 $(\Gamma)_n$ 是衰变宽度。

在一阶微扰近似下，衰变宽度由费米黄金法则给出：$(\Gamma)_n = \frac{(2\pi)}{(\hbar)}\sum_f |\langle f|H_{\mathrm{W}}|n\rangle|^2 \delta(E_f - E_n)$，对于连续末态，求和要换成相空间积分：$(\Gamma)_n = \frac{(2\pi)}{(\hbar)}\int d\Phi_3 \, |\mathcal{M}|^2 \delta(E_n - E_f)$，其中 $d\Phi_3$ 是质子、电子和反电子中

微子的三体相空间，$\mathcal{M}$ 是弱相互作用矩阵元。

这个公式并不是普通势能散射中的简单空间波函数计算，而是由弱相互作用的量子场论矩阵元得到的。中子衰变的标准理论通常称为费米型弱相互作用理论或中子贝塔衰变有效场论。NIST 对中子贝塔衰变的综述也将它描述为重子中的基本带电流弱相互作用过程。

[NIST](https://www.nist.gov/publications/neutron-beta-decay)

2. 低能费米有效哈密顿量

在能量远低于 W 玻色子质量的情况下，可以把弱相互作用写成接触型四费米相互作用。其协变形式大致是 $\mathcal{H}_{\mathrm{W}} = \frac{G_F V_{ud}}{\sqrt{2}} \left[\bar{p}\gamma^{\mu}\left(g_V - g_A\gamma^5\right)n\right]\left[\bar{e}\gamma_{\mu}\left(1-\gamma^5\right)\nu_e\right] + \mathrm{h.c.}$，其中 G_F 是费米常数；V_{ud} 是夸克味混合矩阵元；g_V 是矢量耦合；g_A 是轴矢量耦合；γ^{μ} 和 γ^5 是狄拉克矩阵；h.c. 表示厄米共轭。

中子和质子不是点粒子，所以强相互作用结构会修正耦合常数。通常使用 $\lambda = \frac{g_A}{g_V}$，在允许近似下，中子总衰变率的结构可以概括为 $(\Gamma)_n \propto G_F^2 |V_{ud}|^2 \left(g_V^2 + 3g_A^2\right) f$，其中 f 是由电子和中微子相空间决定的统计率函数。更精确的计算还需要辐射修正、反冲修正、核子结构修正等。现代标准模型理论和有效场论正是用来处理这些高精度修正的。

[OSTI](https://www.osti.gov/servlets/purl/2318474)

3. 非相对论形式：费米跃迁哈密顿量

如果只关心低能中子和质子的自旋结构，可以对核子部分作非相对论展开。弱相互作用哈密顿量可近似表示为 $H_{\mathrm{W}}^{\mathrm{NR}} \sim \frac{G_F V_{ud}}{\sqrt{2}} \int d^3x \left[g_V\, p^{\dagger} n\, e^{\dagger}\nu - g_A\, p^{\dagger}\boldsymbol{\sigma} n \cdot e^{\dagger}\boldsymbol{\sigma}\nu\right] + \mathrm{h.c.}$，第一项是费米型跃迁，第二项是伽莫夫-泰勒型跃迁。符号中的自旋量是二维泡利自旋量，$\boldsymbol{\sigma}$ 是泡利矩阵。在这个近似下，可以把中子看作具有两个内部态的核子：$|n\rangle$, $|p\rangle$，弱相互作用则是把内部态从 $|n\rangle$ 转换到 $|p\rangle$，同时产生电子和反电子中微子。

因此，对于“核子空间 + 电子空间 + 中微子空间”，它确实可以写成一个类似薛定谔方程的跃迁模型：$i(\hbar)\frac{d}{dt}|\Psi(t)\rangle = \left(H_0 + H_{\mathrm{W}}^{\mathrm{NR}}\right)|\Psi(t)\rangle$，不过，这个模型的计算结果仍然依赖于对电子和中微子的量子场处理。把电子和中微子也简单当成普通单粒子波函数，会丢失相对论性、自旋手征性和粒子产生等关键性质。

4. 一个最简的两能级衰变模型

若只想理解衰变的数学机制，可以把中子态和许多末态简化成 $H_{\mathrm{eff}} = \begin{pmatrix} E_n - \dfrac{i(\Gamma)_n}{2} & 0 \\ 0 & E_f \end{pmatrix}$

于是中子分量满足 $i(\hbar)\frac{d}{dt}a_n(t) = \left(E_n - \frac{i(\Gamma)_n}{2}\right)a_n(t)$，其中解为 $a_n(t) = e^{-iE_n t/(\hbar)} e^{-(\Gamma)_n t/(2\hbar)}$，非厄米项 $-\frac{i(\Gamma)_n}{2}$ 不是基本哈密顿量中的真实势能，而是把所有未显式写出的衰变末态消去后得到的有效描述。它表示概率从“中子子空间”流入“质子 + 电子 + 反电子中微子”子空间。

更完整地说，所有末态联合起来形成一个连续谱，而不是一个单独的 $|f\rangle$。正是连续谱导致稳定的指数衰减区间和费米黄金法则。

薛定谔型有效模型能够很好地描述：1. 初态中子振幅随时间的衰减；2. 中子衰变时间与衰变宽度的关系；3. 费米黄金法则；4. 电子能谱和末态相空间；5. 不同自旋跃迁的相对贡献；6. 在适当修正下的角关联和极化关联。

但它不能独立、完整地取代标准模型量子场论，原因包括：1. 电子和中微子是相对论性粒子；2. 衰变改变粒子数和粒子种类；3. 弱相互作用具有手征结构；4. 需要处理反粒子；5. 高精度衰变时间计算需要电弱辐射修正；6. 质子和中子的内部夸克结构会影响矩阵元。

所以最准确的表述是：中子衰变可以用薛定谔型的有效演化方程描述其衰变动力学，但衰变矩阵元本身必须来自相对论量子场论或低能弱相互作用有效场论。

其逻辑链条可以概括为：标准模型弱相互作用 $\longrightarrow$ 费米有效哈密顿量 $\longrightarrow$ 初态-连续末态模型 $\longrightarrow$ 薛定谔型有效方程 $\longrightarrow P_n(t) = e^{-t/(\tau)_n}$。

事实：真正的中子衰变不是严格的两能级系统，因为衰变后的质子、电子和反中微子有无数种不同动量和能量组合。两能级模型只能用来理解“中子态的概率为什么会减少”，不能精确计算真实中子衰变时间。

用薛定谔方程描述衰变的时间演化，通常采用“离散初态 + 连续末态”的有效哈密顿量，得到指数衰减、衰变宽度和费米黄金法则。

中子衰变是弱相互作用导致的粒子种类改变过程，并且产生电子和反电子中微子，严格理论需要量子场论。实际常用的是相对论量子场论或其低能有效理论。

7 中子衰变时间是一个“常数”的意义

说“中子衰变时间是一个常数”，通常并不是说每个中子都能在完全相同的时刻衰变，而是说自由中子的衰变具有稳定的统计规律。对于一个单独的中子，我们无法准确预言它会在什么时候衰变；它可能很快衰变，也可能存活很长时间。具体衰变时刻具有随机性。

物理学中常用“平均衰变时间”来描述自由中子的衰变特征，记作 τ_n，其数值大约为 880 s，也就是大约 15 分钟。平均衰变时间的含义是：如果观察数量足够多的中子，把它们各自从产生到衰变的时间进行统计平均，所得结果会逐渐接近这个数值。它不是每个中子的实际衰变时间。

自由中子经过时间 t 后仍未衰变的比例，可以用指数规律表示为 $P(t) = e^{-t/\tau_n}$。当时间等于平均衰变时间时，仍未衰变的比例约为 e^{-1}，也就是 36.8% 左右；已经衰变的比例约为 63.2%。这说明平均衰变时间反映的是整体统计特征，而不是单个粒子的确定命运。

与平均衰变时间对应的还有衰变常数 λ_n，二者关系为 $\lambda_n = 1/\tau_n$。衰变常数表示一个尚未衰变的中子在单位时间内发生衰变的概率率。只要环境条件不变，这个概率率就保持不变。因此，一个中子已经存活了多久，并不会改变它下一瞬间发生衰变的基本概率。

所以，“常数”的真正意义在于：自由中子的微观衰变规律和统计参数保持稳定，而不是衰变时刻被预先固定。需要注意的是，这一结论主要适用于自由中子；当中子处于原子核内部时，核结构和能量条件可能改变其衰变行为。

“中子衰变时间是一个常数”中的“特殊之处”主要是指它不像长度、质量那样只是一个物体静态具有的普通属性，而是一个描述量子随机过程速度的物理常数。

更准确地说，自由中子的平均衰变时间记为 τ_n，它表示大量处于相同条件下的自由中子，其衰变时间统计平均值会趋近于同一个稳定数值。它不是说每一个中子都会恰好在 τ_n 秒后衰变。

这个常数特殊在于，它同时具有以下几层意义：

第一，它是统计常数。单个中子何时衰变无法确定，只能给出概率；但是大量中子的总体衰变规律非常稳定。存活概率满足 $P_{\text{survive}}(t) = e^{-t/\tau_n}$。因此，个体行为随机，整体规律却精确。

第二，它是无记忆过程的常数。一个中子已经存在多久，不会改变它在下一小段时间内衰变的基本概率率。衰变常数为 $\lambda_n = 1/\tau_n$，在环境不变时保持恒定。这与人或机械的“越老越容易损坏”不同；中子并不存在通常意义上的老化。

第三，它是基本相互作用的结果。中子衰变时间不是人为规定的数字，而是弱相互作用强度、夸克混合、核子内部结构以及衰变末态可用能量耦合后近似等价于基本相互作用的结果。自由中子可发生的过程是 $n \to p + e^- + \bar{\nu}_e$。所以，中子衰变时间把微观弱相互作用的信息浓缩成了一个可测量的数字。

第四，它具有普适性和条件性。在相同物理条件下，任何自由中子都应服从同一平均衰变时间；但若中子被束缚在原子核内，核结合能和核结构会改变衰变是否允许及其速率，此时不能直接套用自由中子的衰变时间。

因此，“常数”的含义是：它是量子随机性、相互作用和稳定统计规律交汇处的一个常数。

8 参考文献

本篇文章的部分推导和理论已经是专业常识，另外的部分假设和联系完全是基于本人的个人研究，所以，本篇文章无参考文献。

另外，本人隆重感谢人工智能，虽然有时候它固执的像个伞兵，但有时候它还是个很棒的个人助手。